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# Creep strength of discontinuous fibre composites

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A unidirectional, discontinuous fibre composite is considered under conditions of steady state creep in the direction of reinforcement. The composite consists of noncreeping, discontinuous, perfectly aligned, uniformly distributed fibres which are perfectly bonded to a matrix obeying a power relation between stress and strain rate. Expressions for the interface stress, the creep velocity profile adjacent to the fibres and the creep strength of the composite are derived. Previous results for the creep strength, $\sigma_c$ obtained for composites of the same type are briefly reviewed and compared with the present result. It is shown that all results reduce to the same general expression

$$\sigma_c = \alpha V_f \sigma_0 \left(\frac{\dot{\epsilon}}{\epsilon_0}\right)^{1/n} \rho^{1+1/n}$$

in which $\rho$ is the fibre aspect ratio, $\dot{\epsilon}$ is the composite creep rate, $V_f$ is the fibre volume fraction, $\sigma_0$, $\epsilon_0$ and $n$ are the constants in the matrix creep law. The creep strength coefficient $\alpha$ is found to be very weakly dependent on $V_f$ and practically independent of $n$ when $n$ is greater than about 6.

## 1. Introduction

It has become well established over the past decade that the resistance to creep at elevated temperatures of a metal matrix can be enhanced by the addition of creep resistant fibres [1-4]. Considerable effort has, therefore, been directed toward the development of mathematical models describing this enhancement in terms of the parameters specifying the constitution of the composite.

Mileiko [5] proposes a model which extends under simple shear of the matrix and with no matrix contribution to the tensile load supported by the model. A small matrix contribution is included in Kelly and Street's model [6] which is based on the assumption of a uniform shear strain-rate in the matrix. McLean [7] employs the same assumption, but derives the creep strength from an energy consideration, and points out that the matrix contribution usually is fairly large. In fact, he concludes that the tensile load is approximately equally divided between fibres and matrix independently of relative volume fractions, as long as flow occurs in the matrix. This is a result of the constraint effect observed in tensile experiments by Kelly and Lilholt [8]. As the matrix deforms plastically, its transverse contraction is constrained by the surrounding fibres which are normally less compliant. In Kelly and Lilholt's experiments this caused a very high apparent tensile stress in the matrix, which disappeared when the fibres yielded, and hence the transverse contractions became the same.

## 2. The model composite

The approach described here is based on a model which we define by a number of assumptions. The model consists of a matrix containing noncreeping, aligned, cylindrical fibres of diameter $d$ and of length $l$. The aspect ratio $\rho = l/d$ is assumed to be sufficiently large that end effects may be ignored. The distribution of the fibres is random in a direction parallel to the fibre axis. In cross-sections normal to the fibre axes the fibres are uniformly distributed so as to form a hexagonal array.

The creep behaviour of the matrix is assumed to be described by the usual power relation

$$\dot{\gamma} = \gamma_0 \left(\frac{\tau}{\tau_0}\right)^n \tag{1}$$

relating the shear strain-rate, $\dot{\gamma}$, to the shear stress $\tau$. The constants, $\gamma_0$, $\tau_0$, and $n$ are related to those obtained in a tensile creep test, $\epsilon_0$, $\sigma_0$, and $m$, by [6]

$$\gamma_0 = \frac{3}{2}\epsilon_0, \tau_0 = \frac{1}{2}\sigma_0, n = m\,. \tag{2}$$

The adhesion between fibres and matrix is thought to be perfect, so that no relative sliding can occur at the interfaces during creep of the composite.

## 2.1. The stress system

In a condition of steady state creep the matrix is assumed to extend at the constant rate, $\dot{\epsilon}$, at points midway between two fibres. Taking $z = 0$ at a fibre centre (Fig. 1) this results in a velocity $\dot{\epsilon}z$ relative to the rigid fibre. Since unlimited interface strength is assumed, the relative velocity must decrease to zero at the interface. This is what causes load to be exchanged between the components by means of the interface stress, $\tau_i$. Previous authors [5-7] all found that the interface stress is nearly constant along the major portion of the fibre length when the stress exponent, $n$, is greater than about 4. Since the tensile load on the fibre is found by integrating $\tau_i$ from the fibre end to the point in question, this results in a nearly linear variation of the tensile load on the fibre along its length.

In discussing the distribution of tensile load between the components, McLean points out that the tensile load on the matrix can be obtained by a similar integration of $\tau_i$ from the fibre centre. For higher $n$-values the tensile load in the matrix consequently rises almost linearly from the fibre centre. McLean further concludes that the tensile load of the composite is roughly equally divided between matrix and fibres irrespective of volume fractions. This is in marked contrast to the case in which the matrix does not yield plastically. Here there is normally a preferential loading of the fibres.

In view of the above it appears that reasonable simple assumptions regarding the stress system can be stated by the following:

1. the tensile stress supported by the matrix varies linearly from fibre centre to fibre end at a constant rate given by

$$\frac{\partial\sigma}{\partial z} = \frac{2\sigma_m}{l} \tag{3}$$

where $\sigma_m$ denotes the greatest tensile stress in the matrix;

2. the tensile load carried by the composite is nearly equally divided between the components at any fibre volume fraction, $V_f$. The creep strength, therefore, merely becomes twice the load supported by the fibres, $\sigma_c = 2\bar{\sigma}_f V_f$. When linear variation of tensile stress in the fibre is assumed, the average stress $\bar{\sigma}_f$ is given by $\bar{\sigma}_f = 1/2\ \sigma_f$, where $\sigma_f$ is the tensile stress at the fibre centre. We shall therefore assume that

$$\sigma_c = V_f\sigma_f = (1 - V_f)\sigma_m\,. \tag{4}$$

## 2.2. The stress transfer

The geometry and the stress system of the model composite are illustrated by Fig. 1. The two

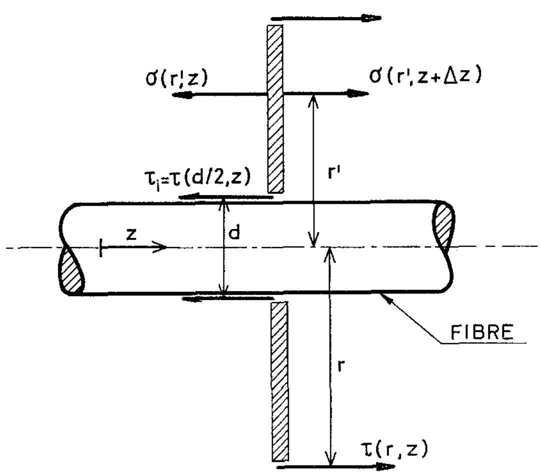


*Figure 1* The stresses acting on a thin circular slice of matrix material.

hatched rectangles represent a thin circular slice of matrix. By integrating the stresses exerted on this slice we can obtain the $z$-component of the total force acting on it. The exact result for a single fibre embedded in the matrix is

$$\left\{2r\tau - \tau_i d + 2\int_{\frac{1}{2}d}^{r} r'\frac{\partial\sigma}{\partial z}\,dr'\right\}\Delta z\,.$$

In a state of mechanical equilibrium the total force is zero, and in an attempt to average we shall represent the equilibrium equation of the model by

$$2r\tau - \tau_i d + 2\int_{\frac{1}{2}d}^{\frac{1}{2}d+h} r\frac{2\sigma_m}{l}\,dr = 0\,. \tag{5}$$

We have employed Equation 3 and further set $h$ equal to half the minimum surface-to-surface distance between fibres in the model composite. Because of the assumed hexagonal distribution, $h$ is therefore written

$$h = \frac{d}{2}\left\{\left(\frac{2\sqrt{3}}{\pi} V_f\right)^{-1/2} - 1\right\}. \tag{6}$$

Using Equations 4, 5, and 6 we find

$$\tau = \frac{\tau_i^* d}{2r} \tag{7}$$

where

$$\tau_i^* = \tau_i - \frac{\sigma_f d}{2l}\frac{V_f}{1 - V_f}\left\{\left(\frac{2\sqrt{3}}{\pi} V_f\right)^{-1/4} - 1\right\}. \tag{7a}$$

## 3. The radial velocity profile

The matrix shear stress is given by Equation 7 and the response to shear stress by Equation 1 in which $\dot{\gamma}$ is related to the relative velocity, $u$, between fibres and matrix through

$$\dot{\gamma} = \frac{\partial u}{\partial r}. \tag{8}$$

Using Equation 2 the rate of increase of $u$ with increasing distance from the interface can, therefore, be written

$$\frac{\partial u}{\partial r} = \frac{3}{2}\epsilon_0\left(\frac{\tau_i^* d}{\sigma_0 r}\right)^n. \tag{9}$$

By integrating $\partial u/\partial r$ with respect to $r$ and employing the condition that there is no sliding at the interface, we find the velocity profile

$$u = u^*\left[1 - \left(\frac{2r}{d}\right)^{1-n}\right] \tag{10}$$

where

$$u^* = \frac{3}{4}\left(\frac{\epsilon_0 d}{n - 1}\right)\left(\frac{2\tau_i^*}{\sigma_0}\right)^n.$$

The reduced velocity, $u/u^*$ is plotted as a function of $r/d$ in Fig. 2 for a number of values of $n$. It is seen that the creep of the matrix is opposed only in the rather close vicinity of the fibres in the model composite, when $n$ is not too small.

Street [9] has observed the velocity profile on the surface of a composite consisting of lead reinforced by phosphor-bronze plates. The surface was initially intersected by straight marker lines and then subjected to creep. A micrograph of the surface after creep is reproduced in Fig. 3. The straight marker lines have been deformed into curved lines giving evidence of a localized shear zone. Street [9] reports a value $n = 14$ for the lead matrix so the slope of the marker lines appears to be less than that of the velocity profiles of the model. This is undoubtedly a consequence of the idealized shear stress variation of the model, since a power relation causes the creep rate to depend quite strongly on stress. However, one might

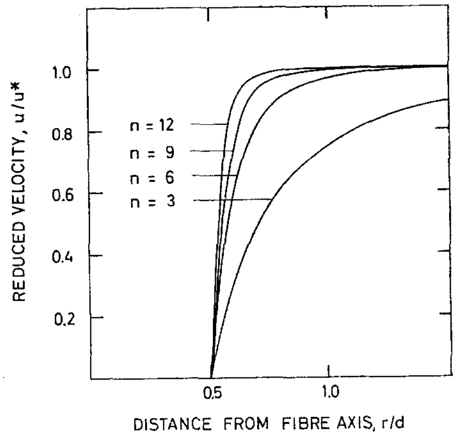


*Figure 2* Profiles of reduced velocities, $u/u^*$, adjacent to the fibre-matrix interface, as predicted by Equation 10.

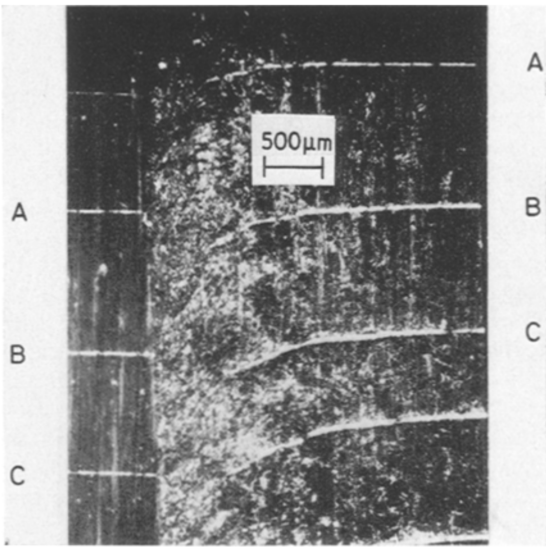


*Figure 3* Optical micrograph of the tensile creep of lead embedding a noncreeping phosphor-bronze plate. The originally straight marker lines labelled A, B, C are just visible in the shear zone (Street [9]).

indeed expect the localization of shear to be less pronounced when the reinforcing members are plates rather than fibres.

## 4. The composite creep strength

The creep rate in the matrix was seen in the foregoing section to be disturbed only close to the fibre-matrix interface. In deducing the shear stress exerted on the interface we shall therefore disregard the presence of other fibres and employ the relationship

$$u(r, z) \to \dot{\epsilon} z \text{ as } r \to \infty \tag{11}$$

which applies when $n$ is greater than 1. By combining this and Equation 10 we find

$$\tau_i^* = \beta\sigma_0 \left(\frac{\dot{\epsilon}}{\epsilon_0}\right)^{1/n} \left(\frac{z}{d}\right)^{1/n} \tag{12}$$

where

$$\beta = \frac{1}{2}\left(\frac{4}{3}\right)^{1/n} (n-1)^{1/n} .$$

Equation 12 has a formal similarity to Kelly and Street's result for the interface stress, but in the present case $\beta$ is not a function of volume fraction.

According to assumption 2 (Section 2.1) we can express the creep strength as $\sigma_c = \sigma_f V_f$, where $\sigma_f$ is determined by the force balance

$$\sigma_f = \frac{4}{d}\int_0^{l/2} \tau_i \, dz . \tag{13}$$

The interface stress is given in terms of $\sigma_f$ by Equations 7a, and 12, so by substituting for $\tau_i$ in Equation 13 we find the composite creep strength

$$\sigma_c = \alpha V_f \sigma_0 \left(\frac{\dot{\epsilon}}{\epsilon_0}\right)^{1/n} \rho^{1+1/n} \tag{14}$$

where

$$\alpha = \frac{(1-V_f)}{1 - V_f\left(\frac{2\sqrt{3}}{\pi} V_f\right)^{-1/4}} \left(\frac{2}{3}\right)^{1/n} \frac{n(n-1)^{1/n}}{n+1} .$$

In the next section it will be seen that the creep strength coefficient, $\alpha$, is approximately constant.

## 5. Discussion and conclusions

The models developed by Mileiko, Kelly and Street, McLean, and the author all represent what might be called a perfect composite: fibres and matrix are fully adhering, the fibres are noncreeping, chemically stable, perfectly aligned, and evenly distributed. It is, therefore, interesting to compare the predictions these models give for the creep strength. It turns out that a comparison may be made very conveniently because a remarkable similarity exists between the results. By introducing the notation used in this paper and rearranging we can express the creep strength by Equation 14 for all models with expressions for the coefficients given by:

(McLean)

$$\alpha_1 = \frac{1-V_f}{V_f}\left\{2\left(\frac{2\sqrt{3}}{\pi} V_f\right)^{-\frac{1}{2}} - 2\right\}^{-\frac{n+1}{n}}$$

(present model)

$$\alpha_2 = \frac{(1-V_f)}{1 - V_f\left(\frac{2\sqrt{3}}{\pi} V_f\right)^{-1/4}} \left(\frac{2}{3}\right)^{1/n} \frac{n(n-1)^{1/n}}{n+1}$$

(Kelly and Street)

$$\alpha_3 = \frac{n}{2n+1}\left[\frac{3}{2}\left(\frac{2\sqrt{3}}{\pi} V_f\right)^{-\frac{1}{2}} - \frac{3}{2}\right]^{-1/n} + (1-V_f)\,\rho^{-\frac{n+1}{n}}$$

(Mileiko)

$$\alpha_4 = \left(\frac{n-1}{1 - V_f^{(n-1)/2}}\right)^{1/n} \int_0^1 \left\{z^{-n} + (1-z)^{-n}\right\}^{-1/n} dz$$

In McLean's coefficient, $s$ has been replaced by $2h$ where $h$ is given by Equation 6. The expressions for the coefficients have been computed as functions of $V_f$ for four $n$-values ranging from rather small, 3 and 6, to rather greater, 9 and 12. Kelly and Street's coefficient is shown for $\rho = 50$ (dashed curve) and $\rho = \infty$ (solid curve). Although the $\rho$-dependence is quite significant at small $V_f$-values it is seen to be fairly unimportant when $V_f$ is greater than about 20%.

It is evident from Fig. 4 that all creep strength coefficients are nearly independent of $n$ when $n \gtrsim 6$. The variation of $\alpha$ with $V_f$ is also seen to be quite small so it seems reasonable simply to represent the coefficients by

$$\alpha_1 = 1.5$$
$$\alpha_2 = 1.2$$
$$\alpha_3 = 0.4$$
$$\alpha_4 = 0.3$$

which roughly corresponds to the $\alpha$-values at

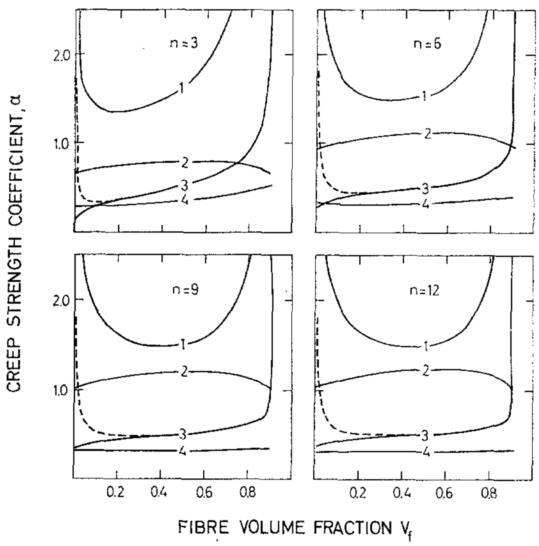


*Figure 4* The creep strength coefficients, $\alpha_i$. Index i refers to each curve as indicated (1, McLean, 2, present model, 3, Kelly and Street, 4, Mileiko). Kelly and Street's coefficient has been computed for $\rho = 50$ (dashed curve) and $\rho = \infty$ (solid curve).

$V_f = 50\%$ and $n = 12$. The only important point at which the four models fail to agree is therefore in predicting the creep strength coefficient. However, the predicted coefficients seem to increase with increasing assumed matrix contribution, this could explain part of the disagreement. Excellent agreement is seen between $\alpha_1$ and $\alpha_2$ which both take full account of the matrix and the general expression

$$\sigma_c = \alpha V_f \sigma_0 \left(\frac{\dot{\epsilon}}{\epsilon_0}\right)^{1/n} \rho^{1+1/n}$$

where $\alpha$ is approximately constant and near unity seems to be very well supported by theory, since it emerges from all four treatments. Once the general expression has been experimentally verified, the decision between the various models can, therefore, be made simply by measuring the creep strength coefficient.

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## References

1. A. KELLY and K. N. STREET, *Proc. Roy. Soc. Lond.* **A328** (1972) 267.
2. A. KELLY and W. R. TYSON, *J. Mech. Phys. Solids* **14** (1966) 177.
3. K. N. STREET, NPL Conf., November 1971, Paper 3.
4. B. A. WILCOX and A. H. CLAUER, *Trans. Amer. Inst. Metall. Petrol Eng.* **245** (1969) 935.
5. S. T. MILEIKO, *J. Mater. Sci.* **5** (1970) 254.
6. A. KELLY and K. N. STREET, *Proc. Roy. Soc. Lond.* **A328** (1972) 283.
7. D. MCLEAN, *J. Mater. Sci.* **7** (1972) 98.
8. A. KELLY and H. LILHOLT, *Phil. Mag.* **20** (1969) 311.
9. K. N. STREET, private communication (1973).

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